Use of Geometry Knowledge During Problem Solving: The Instructional Effect of Two Strategies

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The application of mathematical conceptual knowledge in the solving of problems has been of continuing interest to researchers and classroom teachers. Towards this end, research evidence is providing some insights as to the role of mathematics schemas in the solution process. Little research effort, however, has been invested in the examination instructional methods that will enhance the activation and use of such schemas by students.

In this study we report the results of two studies which attempt to generate data relevant to the effect of instructional strategies on the activation and use of previously learned schemas in junior high school geometry. Two forms of training derived from our earlier studies of geometry problem solving were developed. The first, *Generation* training emphasised searching memory to access information that is related to the problem. *Management* training directed students' attention to planning and controlling accessing of information. No effects on training or transfer items were observed following Generation training. Management training improved the near and far transfer performance of both high- and low-achieving students.

Student difficulties that are associated with the use of domain-specific knowledge has been the concern of research in recent times. Failure to access and use relevant knowledge spontaneously is a major problem for teachers and students. Bereiter and Scardamalia (1986) observed students' access problems in expository writing. Bransford, Sherwood, Vye and Reiser (1986) regarded difficulties with knowledge access and use as a major cause of poor performance. In a mathematics classroom, the successful outcome of an activity is governed by the ability of the student to activate and make appropriate use of previously learned conceptual knowledge. The spontaneous use of available knowledge is particularly critical during problem solving attempts where students have to work individually and cue themselves to draw upon knowledge relevant to the problem.

In an examination of major processes underlying successful solution outcomes in the domain of high school geometry, Lawson and Chinnappan (1994) found that, compared to their less successful peers, the more successful students activated available knowledge more often and used these items of knowledge as cues in order to produce even more information . The category of events that were associated with this type of information processing was labelled as *Generative Activities*. These were designed to aid students in the production of new information through the more detailed analysis of information presented in the problem statement and wider use of that information in directing the search for other problem-relevant knowledge. A second major category of cognitive events that was reported in the above study, namely, *Management Activities* was concerned with how students directed their search in the problem space. The results showed that the high-achieving (HA) students invoked greater numbers of both Generative and Management events than the low-achieving (LA) students. These differences were most apparent when students attempted to solve the more difficult problems. Study of individual transcripts showed that students activated Management events when generation of information along a certain path did not lead to the goal state.

We argue that both Generative and Management activities have a direct effect on students' activation and use of mathematical knowledge during problem solving. Students who are actively generating information and managing their problem-solving moves can be expected to a) search a wider network of their knowledge base and b) reflect upon the effect of their generational moves and, if necessary, make a decision about shifting the focus so that they seek a solution along an alternative path. Thus, both generation and management moves can be seen as being instrumental in initiating and maintaining students' effort at accessing and making productive use of previously learned knowledge of geometry. This line of reasoning motivated us to undertake the present studies in which we explore the effects of instructing students in use of Generation and Management strategies on accessing and using geometry knowledge during the solution of problems that were similar to those employed in the training phase and, on problems that are different from those used in training phase. The studies were expected to generate evidence to support the hypotheses that the treatment group would perform significantly better than the control group on transfer problems and in the frequency of information-production moves made during solution search. We also expected that students in the low-achieving group would show more improvement in these two areas than those in the high-achieving group.

DESIGN

The design of both studies was similar. A period of about two weeks regular class teaching on the topic was followed by a training period lasting two class sessions. Post testing occurred in the subsequent class sessions. All training was carried out through use of prepared booklets.

STUDY 1

Subjects

The subjects were 27 Year 10 students in a high school in Adelaide. The class was ranked second out of the seven classes streamed for mathematics on the basis of Year 9 performance. Students in this class would be expected to pursue the study of mathematics into their final year of secondary schooling. The top third of the students, as indicated by class test performance, were assigned to the HA group, with the remaining students forming the LA group.

Procedure and materials

The content for the classes in this study involved right-angled triangle problems using the sine, cosine and tangent ratios and was carried out over a three week period. The first 7 lessons involved regular class work. The first training session was carried out in lessons 8 and 9 and was followed by the first post-test in lesson 10, with a second training session in lessons 14 and 15 after further teaching. A second post-test was held in lesson 16. The results reported here are for the combined scores for both post-tests.

All training material was provided in workbooks using a worked example, non-worked example sequence of problems. Workbooks were distributed according to a stratified random assignment pattern, with students being ranked in pairs according to level of performance on previous class test and each member of a pair being assigned at random to either the Experimental or Control group. The workbooks of the two groups differed in that extra strategic information was provided in the worked examples for the experimental group. Otherwise the layout of the two booklets was designed to appear as similar as possible.

In the Generation training the extra material provided in the workbooks emphasised the importance of careful analysis of the problem statement, searching of memory for previous experience with the problem type, and the use of cues in the problem statement and diagrams to access other related information. In the worked examples the text required the student to attempt to solve the problem by following a series of steps. Once the student had made the solution attempt the following page provided the text of a solution protocol which highlighted the use of the elements included in the training.

The post-tests used in both studies were composed of two classes of items. Approximately half of the items were close relations of the items used in the training workbooks. These are labelled as Training Items. The remaining items, the Transfer Items, required students to apply learned rules to trigonometry problems in new contexts. These items could, for example, require the construction of lines in order to identify right-angled triangles which had to be used in application of the appropriate ratio, or the construction of a line whose length had to be calculated in order that the target quantity in another triangle could be found. We argue that these items measure significant and valued near, within-domain transfer.

All answers to these problems were subjected to a second analysis in which subjects' solution transcripts were examined carefully to determine the frequency of correct generational moves (Generation score). A generation move refers to a step in the solution process during which an item of new problem relevant information was produced. This analysis was expected to provide a second measure of the extend of knowledge use during the solution attempt.

Results

The scores on Training and Transfer Items were analysed in separate 2(Level: HA or LA) x 2(Group:Experimental or Control) analyses of variance. The only statistically significant effect found in the post-test scores following generation training was a Level effect on the Training Items, where HA students outperformed their LA counterparts. While the LA students who received training scored higher than their Control counterparts the differences were not statistically significant. A similar 2 x 2 ANOVA conducted on the Generation scores revealed no significant effects.

Table 1: Post-test means (standard deviations) for Generation training study - Study 1

LEVEL	GROUP	ITEMS		GENERATION SCORE
		Training (Max = 7)	Transfer (Max = 12)	
Low Achieving	Experimental	4.13 (0.64)	4.75 (1.28)	66.31 (5.27)
	Control	3.00 (1.20)	3.75 (2.49)	49.69 (23.57)
High Achieving	Experimental	4.67 (1.16)	5.00 (1.00)	71.50 (18.03)
	Control	5.20 (1.30)	6.40 (2.88)	76.10 (22.71)

STUDY 2

Subjects

The subjects were 26 Year 10 students in a high school in Adelaide. The class was ranked third out of the seven classes streamed for mathematics on the basis of Year 9 performance. Students in this class would be expected to pursue the study of mathematics into their final year of secondary schooling. The top third of the students, as indicated by class test performance, were assigned to the HA group, with the remaining students forming the LA group.

Procedure and materials

The procedure and materials used in this study were the same as used in Study 1 except for the replacement of Generation training with Management training. Students in this study completed the same cycle of lesson, training and post-tests involving the same lesson content , training items and post-test items as used in Study 1. The information provided in instructional booklets stressed the importance of careful planning of a solution path, the checking of calculations, and reviewing of the solution once it had been generated. Again the post-test items were comprised of Training and near Transfer items.

Results

The means and standard deviations for post-test performance in Study 2 are given in Table 2. No significant differences were observed on the Training Items. On the Transfer Items there was a significant Group effect reflecting a substantial advantage of the Experimental Group on these items. The lack of the expected interaction effect indicated that the advantage conferred by Management training applied to both HA and LA students. The impact of the training effect is shown in Table 2 by the fact that the post-test score of the LA students in the Experimental group was slightly higher than that of the HA students in the Control group. The advantage conferred by the training is a substantial one for both HA and LA students. As for scores on frequency of generation moves (Generation score), there was a significant Level main effect $F(1,19) = 5.55$, $p<0.05$, $MSe = 386.95$, with the HA students showing greater frequency of Generation than the LA group. The significant Group main effect, $F(1,19) = 8.21$, $p<0.01$ indicated that the Experimental students made more Generation moves than did Control students. The Group by Level interaction effect for Generation score was not significant.

Table 2: Post-test means (standard deviations) for Management training study - Study 2

LEVEL	GROUP	ITEMS		GENERATION SCORE
		Training (Max = 7)	Transfer (Max = 12)	
Low Achieving	Experimental	5.00 (1.31)	6.88 (2.64)	82.25 (17.92)
	Control	3.75 (1.17)	3.88 (3.27)	54.88 (26.42)
High Achieving	Experimental	5.00 (0.82)	9.75 (0.50)	101.75 (6.70)
	Control	5.67 (0.58)	6.00 (2.00)	77.67 (6.43)

DISCUSSION

The principal concern of Study 1 was to examine the claim that training students in the use of Generation strategies will significantly improve their accessing and use of geometry knowledge. It was hypothesised this improvement will manifest itself on students' problem solving performance and the number of generation moves made during search for solution. Results showed that the LA students when instructed in the use of Generation strategies tended to do better in the solution of training and transfer problems than their peers who were not given a similar treatment, although this difference in the solution outcome was not statistically significant. However, the improved performance of the LA students in the experimental group was interesting in light of the higher scores obtained by this group in the analysis of generation moves. Generation training seems to have helped the LA students engage in a greater number of generation moves which has the potential to allow them to make greater use of available geometry knowledge during the solution attempt.

Training in Generation strategies required students to think of ways of using particular information. Diverting the solver's attention in this way could be argued to assist students in their efforts to recall strands of information that are functionally linked to that information. Even though production of new information may not necessarily result in achieving the problem goal, it nevertheless constitutes an important precondition. Any improvement in the quantity and quality of information that the solver is able to access and use within a problem-solving context, has the potential to facilitate moves towards attaining the problem goal. Generation training appear to facilitate students' attempts to 'go beyond the information given' (Bruner, 1973).

Study 2 was designed to examine the effect of instruction in the use of Management strategies on use of knowledge during the solution attempts. A two-way analysis of variance showed that there were significant Group main effects for scores in the transfer problems and on the number of generation moves made during solution attempts. In both cases, the treatment group performance was superior to that of the control group. When students had to plan their moves, they had to identify given information and utilize it to generate new information in a manner that would get them closer to the problem goal. It could, therefore, be argued that planning imposed direction and control over the path of generation. At the same time, the planning process forced the solver to make optimum use of information that was identified and information that was generated. The need to trace a path before attempting to carry out any computational work may have resulted in the solver having to explore. Such an activity could increase the amount of new information generated. One would, therefore, expect the treatment group to engage in greater number of generation moves than the control group. The effect of these activities was also reflected in the significantly higher number of correct answers obtained by the treatment group in the posttest. One conclusion that could be made from the above results of Study 2 is that instruction in Management strategies improve problem-solving performance by facilitating purposeful information generation.

One of the predictions made in relation to Management strategies was that the instruction would improve the performance of the low-achieving students on the transfer problems to a greater degree than that of the high-achieving students because it was felt that students who are in the high-achieving group would already be accessing greater amount of available knowledge through some of the management processes. The lack of expected interaction between achievement level and group membership indicated that both the high- and low-achieving students showed significant improvement as a result of the instruction. This shows that there was room for improvement in the case of the high-achieving students.

According to the view developed by Prawat (1989), reflective behaviour plays an important role in mediating access to potentially available information. In the present study Management instruction encouraged students to access and use knowledge which was directed by a plan of action, and assess the effectiveness of that plan. In doing so, these students appear to engage in reflective behaviour.

Our findings involving Management instruction also have implications for the debate concerning the role of general strategies in facilitating learning and problem solving in mathematics (Lawson, 1990; Sweller, 1990). At the core of this debate is the assertion that instruction in general problem solving strategies is not an effective way to improve students' learning of mathematical schemas. Our results show that teaching students a Management strategy that can be argued to be one that has wide application across the domain of mathematical problem solving does significantly aid students controlled accessing and exploitation of knowledge during problem solving.

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